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EXPLICIT SCHEMES OF THE LOCATION METHOD IN THE PROBLEM OF SUPERSONIC FLOWS AROUND A BLUNT BODY

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EXPLICIT SCHEMES OF THE LOCATION METHOD IN THE PROBLEM OF SUPERSONIC FLOWS AROUND A BLUNT BODY

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ABSTRACT. Various schemes of the location method in computing supersonic flows around a blunt body are discussed and compared citing references. Indirect methods indicated low accuracies in the computations and a comparison of computations using the direct method for all three schemes of the integral relation method, the line method, and the location method indicated good concordance of results obtained.

1. The difficulties in computing the steady supersonic flow around two- /514 dimensional blunt bodies stem from the fact that what is being solved is a system of mixed equations, as well as of detailed parameters determining the unknown boundary of the reference region in advance. These difficulties increase significantly with increase in the number of nodes in the reference grid, particularly when passing to the computation of space currents.

From this point of view, location methods have many advantages. In these methods time, t, is introduced as an independent variable, and the solution of the stationary problem is sought as the limit in the nonstationary problem when $t \to \infty$. The problem can be described by a system of equations of the hyper- /515 bolic type. Therefore, despite the increase in the number of independent variables, the solution is simpler than that obtained for equations of the mixed type. An increase in the number of space variables will not complicate the solution algorithm, in principle, but the difficulties in the main are those stemming from the increase in memory and machine time required.

Various schemes of the location method are described in references [1-4], for example. These schemes are still far from perfected, and this is particularly so in the case of three space variables. There are two factors, basically, that determine the improvement in the effectiveness of location methods, while retaining necessary accuracy:

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* Numbers in the margin indicate pagination in the foreign text.

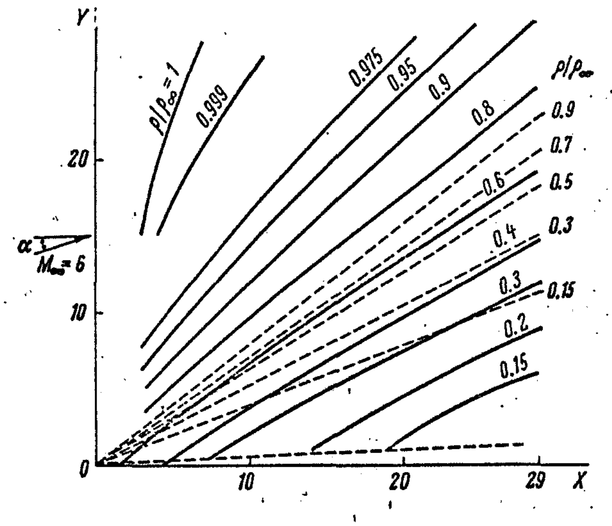

Figure 1.

(a) the possibility of making the computations using an approximate, specified coverage of initial data assigned for t = 0;

(b) reduction in the time required for the computation of the variant.

The first factor is attributable to what follows.

Widely used today is a gradual transition from an available variant to an unknown one, using the parameters of the problem (shape of the body, Mach number, angle of attack, and the like). This procedure, while yielding many subsidiary results, takes a great deal of time, and is unsatisfactory. It is used principally because there is no proof of the existence, or singleness of a solution, and can be explained, in practice, by the fact that very often it is impossible to obtain a solution from the approximate, specified initial data (that is, not from an intermediate solution). It is desirable to create methods and means of making the computation from the approximate initial data, without requiring the obtaining of a complex algorithm for these data, or using another, even more approximate method.

Reduction in variant computation time is associated with the selection of computational schemes such that the time step, τ, can be increased, or with the selection of schemes with an optimal number of operations per time step. The fact is that straightforward explicit schemes with order of accuracy $O(\tau)$ (scheme [5], for example) are either too approximate, or location takes place too slowly when these schemes are used. Schemes of an order of accuracy $O(\tau^2)$ (in [4], for example) provide a more rapid location, but the volume of computations per step is greater by a factor of 6 to 8 (in the case of two space variables) and greater by a factor of 12 to 14 (for three space variables) than in the scheme in [5]. Also to be remembered when seeking optimal schemes is the accuracy of the final result, something that will depend on scheme selection.

2. Results in [3] indicate that the scheme in [5] will yield approximate results in the case of a direct calculation ("diffuse" shock wave). We used the method in this latter to compute the Prandtl-Meyer flow (reference grid 30 x 30 nodes), the flow around a wedge (30 x 116 nodes), and around a flat step (58 x 50 nodes). The computations confirmed the conclusions with respect to the ineffectiveness of this method arrived at in [4]. Compared in Figure 1 are the density distribution, obtained from the computation made for the Prandtl-Meyer flow, and the exact density distribution (dashed). The disturbed region encompasses virtually the entire region in which the computation was made. The shock wave front is greatly diffuse in the case of the flow around the wedge. Practically speaking, the position of the sonic point beyond the step angle is not dependent on time t in the location process, but with decrease in τ (that is, with increase in the coefficient of artificial viscosity) moves /516
downstream from the angular point, and is associated, therefore, with the introduction of viscosity. Figure 2 shows the step velocity distribution at the end of the computation (complete location of the flow was not obtained, and this was true in [3] as well). The dashed curve is the plot of the results of the computation made using the integral relations method obtained in [6].

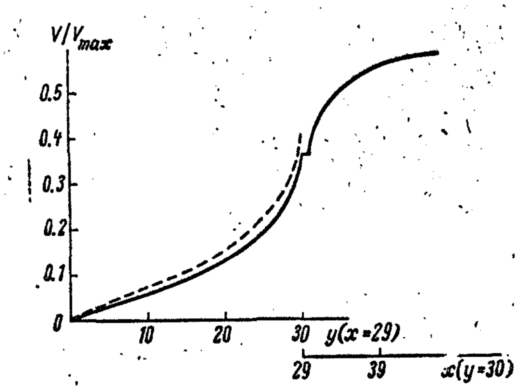

Figure 2.

3. We used the location method to investigate a broader class of schemes in which the shock wave was given as a discontinuity and was the boundary of the region for which the computation was to be made.

The method proposed in [4] was used.

The following schemes were used to compute the interior points of the region when the transition was made to a new time layer:

(1) scheme (L) with artificial viscosity depending on the parameter α:

$$\overset{k+1}{g}_{m,n} = \alpha \overset{k}{g}_{m,n} + (1-\alpha)^{1}/_{4}[\overset{k}{g}_{m+1,n} + \overset{k}{g}_{m-1,n} + \overset{k}{g}_{m,n+1} + \overset{k}{g}_{m,n-1}] + (g_t')^{k}_{m,n}\tau; \quad (1)$$

when $\alpha = 0$ the scheme converts to that in [5];

(2) scheme (L - W):

$$g_{m,n}^{k+1} = g_{m,n}^{k} + (g_t')_{m,n}^{k}\tau + (g_{tt}'')_{m,n}^{k}\frac{\tau^2}{2}. \qquad (2)$$

Here the subscripts correspond to the number of space variable nodes, the superscripts to time variable nodes. The prime with the subscript denotes a partial derivative. The magnitudes of g_t' are determined through the main system of equations, and the second derivatives, g_{tt}'', by the differentiation of the main system with respect to all independent variables. Central differences were used to approximate the derivatives with respect to the space variables.

4. There are two reasons for the oscillations that can occur in the internal field during the solution of the problem: instability of the finite-difference schemes; and the physical process of location. The correct selection of step τ will eliminate the first. The second arises as a result of shock wave motion. Because of the approximate assignment of the initial field, the shock wave will move to and from the body during the computation. This oscillation is damped (Figure 3), and causes the internal field to oscillate. The compressed gas waves thus formed are reflected from the body, return to the shock wave, and can destroy it. The shock wave becomes sawtoothed, and the computational accuracy is reduced.

Introduction of artificial viscosity ($\alpha \neq 1$) significantly reduces the amplitude of wave oscillations, or even makes the process aperiodic (when $\alpha = 0$) (see Figure 3). At the same time the accuracy of the final results decreases. For example, the distance δ_0 of the shock wave from the sphere when $M_\infty = 10$ and $\alpha = 0$ cannot be determined with any more accuracy than 12%. The loss of accuracy in the results in [3] are attributable primarily to the introduction of artificial viscosity ($\alpha = 0$) in the computation of internal field points, and not only by the presence of the diffuse shock wave front.

Accordingly, the scheme in [5] rapidly damps the oscillations in the internal field, but yields more approximate results. It is therefore desirable to gradually increase the parameter α during the computation.

Scheme (L - W) is highly accurate when the oscillation process is relatively weak, but the volume of computations at the point for which they are being made is great.

The procedure of "freezing" the position of the shock wave can be used to avoid the onset of oscillations in the internal field attributable to shock /517

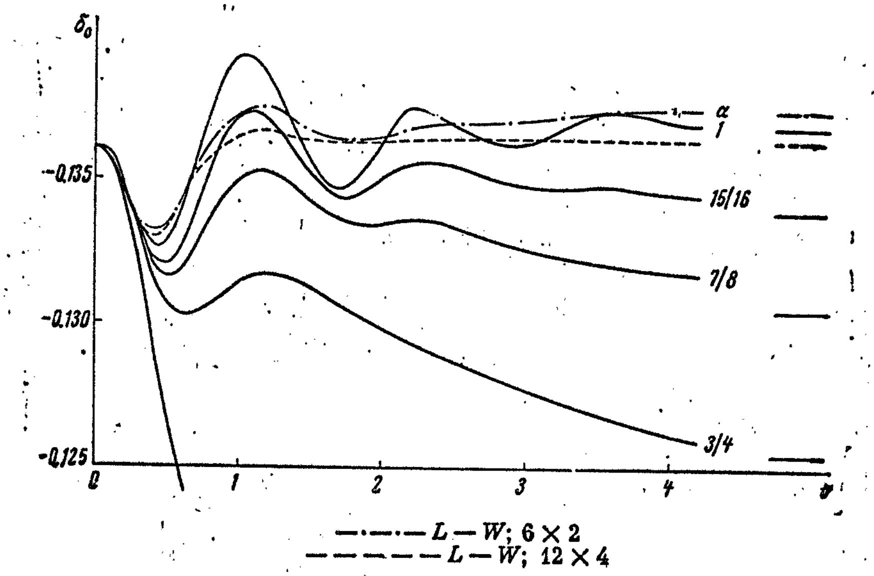

Figure 3.

TABLE 1. SPHERE, $M_\infty = 10$.

Scheme	Number of nodes	δ_0	ρ'_0/ρ_∞	ρ/ρ_∞ on the body when $y = 0.75$	Bernoulli constant in the flow field
$\alpha = 0$	6×2	—0.1004	6.762	3.609	0.9846—1.045
$\alpha = 0$	12×4	—0.1136	6.458	3.335	0.9391—1.023
$\alpha = 0$	24×23	—0.1143	6.524	3.398	0.9520—1.025
$\alpha = {}^3/_4$	6×2	—0.1245	6.312	3.269	0.9996—1.019
$\alpha = {}^7/_8$	6×2	—0.1307	6.224	3.220	0.9958—1.014
$\alpha = {}^{31}/_{32}$	6×2	—0.1355	6.166	3.205	0.9992—1.010
$\alpha = 1$	6×2	—0.1370	6.151	3.208	0.9990—1.009
$L - W$	6×2	—0.1375	6.148	3.197	0.9981—1.009
$L - W$	12×4	—0.1364	6.150	3.198	0.9982—1.004
Integral method:					
scheme I	II appr	—0.1360	6.153	3.18*	
scheme III	1×5	—0.1354	6.153	3.08*	

Note: *data obtained by extrapolation.

wave motion. The wave, at some time interval δt (Figure 4), is taken as fixed, and its position is what it would be at the time the interval begins. The velocity at which the wave is moving is taken as equal to zero. Once the field is stabilized, this flow is equivalent to the flow in a duct, on one wall of which (on the body) the no-flow condition is satisfied, but in which gas is supplied through the other (the shock wave) in accordance with a law corresponding to the Rankine-Hugoniot conditions. The length of the "freezing" interval should be such that the oscillations caused by shock wave motion are damped in the internal field (the number of steps in an interval is 20 to 30, approximately). "Freezing" results in more rapid location, particularly when the initial position of the wave differs little from the true position, and the initial internal field is given as approximate. The solid line in Figure 4 shows wave motion without "freezing," the dashed line that with "freezing." The dash-dot curve corresponds to the limit.

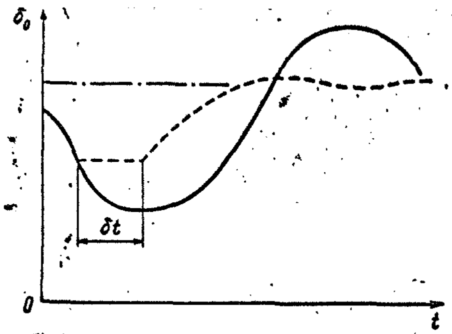

Figure 4.

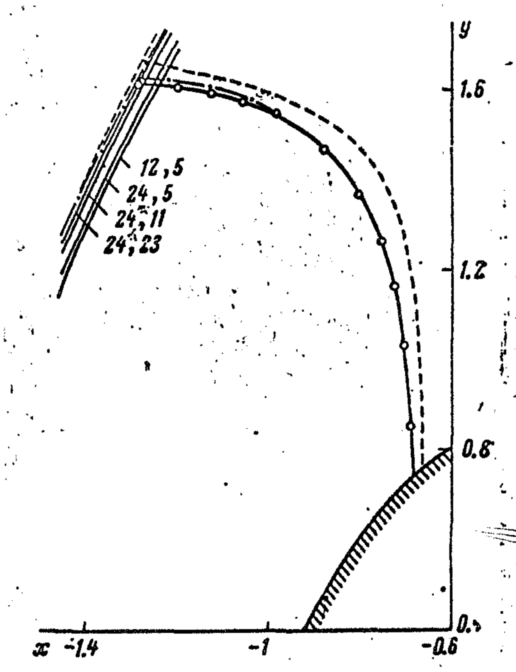

Figure 5.

Distribution of parameters between wave and smooth body is close to linear for large M_∞, and this corresponds to the method we adopted for defining the reference field. The reference field is assigned more and more approximately with reduction in M_∞, and the amplitude of the oscillation of the shock wave increases. "Freezing" effectiveness increases accordingly. This procedure was more effective at small supersonic M_∞ numbers. /518

5. Reference [7] pointed out that computations of the flow around blunt bodies yield results with a pressure distribution error in excess of 10%. (It is known that pressure on a body is a conservative characteristic, and can be computed quite accurately.) The low accuracy can be explained by the use of indirect methods. The comparison made in [8], as well as in this paper, is indicative of the good concordance of the results obtained by direct methods. The comparison was made for all three schemes of the integral relation method, for the line method, and for the location method. Flows over spheres and cylinders in the range of change in M_∞ numbers from 2 to 10 were compared. The concordance in the axisymmetrical case was better than in the flat case, and better for large M_∞ numbers than

for lesser ones. The law of distribution of parameters between body and shock wave was close to linear for large M_∞ numbers. The result was that even when the number of internal field nodes was small (6 x 2, for example), quite accurate results were obtained, and the solution was relatively insensitive to increase in the number of nodes in the grid (see the distance δ_0 between body and shock wave, for example, or the ratio of the density at the stagnation

TABLE 2. CYLINDER, $M_\infty = 3$.

Scheme	Number of nodes	δ_0	Bernoulli constant in the flow field
L-W	12x5	-0.6726	0.9475-1.037
"	24x5	-0.6791	0.9573-1.010
"	24x11	-0.6901	0.9981-1.007
"	24x23	-0.6953	0.9993-1.006
Method of straight lines	4 rays	-0.6988	
Integral method scheme I	III approx.	-0.703	

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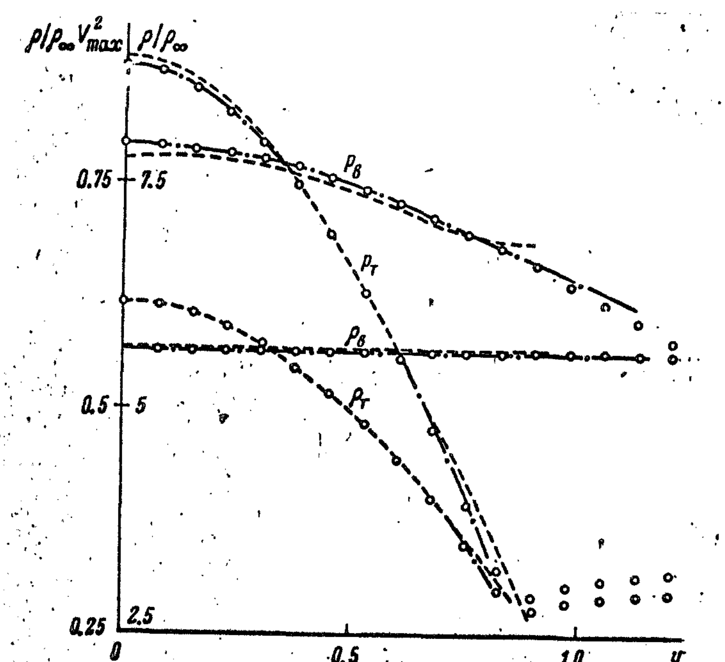

Figure 6.

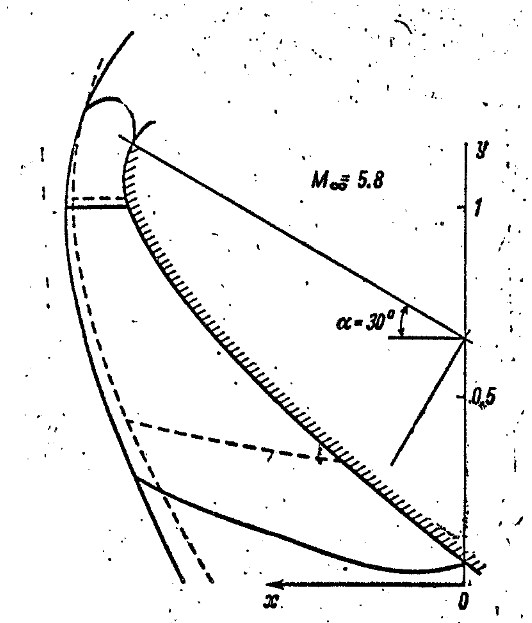

Figure 7.

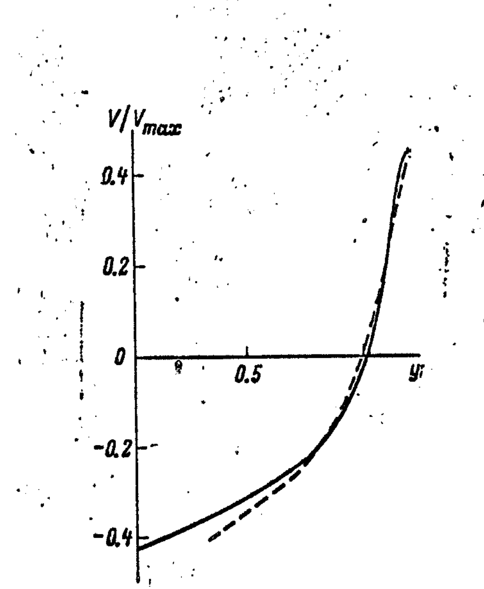

Figure 8.

point to the density in the incoming flow ρ_0'/ρ_∞, Table 1). The number of reference points must be increased in order to reduce the M_∞ numbers (particularly in the case of a plane flow). The dependence of the position of the wave on the number of internal field nodes will be seen from Table 2 and Figure 5. The results of the computations are very close to the data A. P. Bazzhin and I. F. Chelysheva received by using the line method, and the integral relations method (schemes I and III from [8]). The differences in the positions of the shock waves were not in excess of 1%. The line on the cylinder obtained by O. M. Belotserkovskiy [9] differs somewhat from that obtained using the location method and the line method (Figure 5). Figure 6 compares pressure and density on the surface of a cylinder and the shock wave when $M_\infty = 3$, from data obtained using various methods. The dashed curves in Figures 5 and 6 are for the III approximation when the integral relations method was used (scheme I) [8], and the dash-dot curves are for the lines and small circle method, and the location method. /520

6. The flow around a profile with great change in surface curvature was computed. The profile equation used was $y_0 = \pm[1/2\sqrt{(x_0+1)}-0.04(x_0+1)^2]$. The results of this computation (the solid curves) are compared with the data obtained by A. P. Bazzhin in 1963, using the first approximation of the integral relations method (the dotted lines), in Figure 7 and 8. Shock wave positions differ about 10%. The integral method yielded more precise parameters on the body than it did wave position, except in the vicinity of the sonic points. The y coordinate of the stagnation point differs by 2%.

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